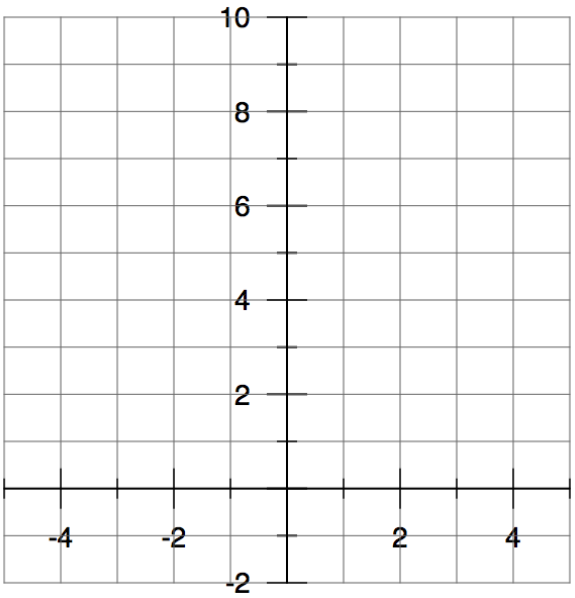
Reflections (Part 2)

These notes are intended as a supplement to section 3.2 (p. 178 – 183) in your workbook. The topics discussed here are not included in the workbook.

Even Functions

A function is called even if $f(-x) = f(x)$ for all values of x in its domain. The graph of an even function is symmetric with respect to the y-axis. In other words, for every point (x, y) on the graph, there is a corresponding point $(-x, y)$.

Example 1
If $f(x) = x^2$, then draw the graphs of $y = f(x)$ and $y = f(-x)$ on the same axes.



To test if a function is even, simply replace x with $-x$ and simplify. If the result is the same as the original function, then the function is even.

Example 2
Is $f(x) = x^4 + x^2 - 3$ an even function?

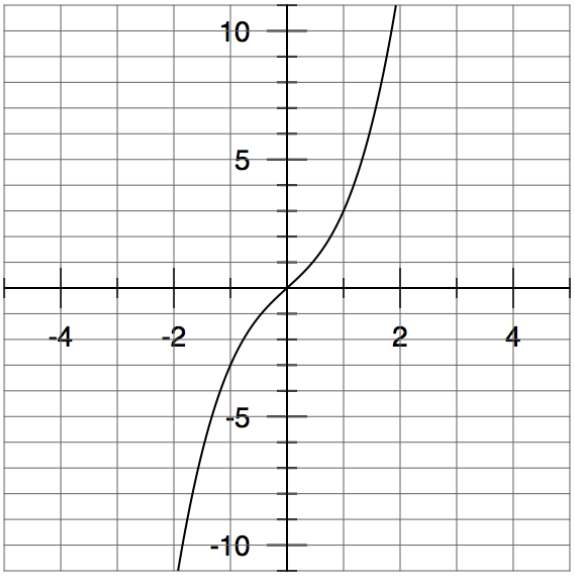
Note: Any polynomial function involving only even powers of x will be even.

Odd Functions

A function is called odd if $f(-x) = -f(x)$ for all values of x in its domain. The graph of an odd function is symmetric with respect to the origin. In other words, for every point (x, y) on the graph, there is a corresponding point $(-x, -y)$.

Example 3

The graph of $f(x) = x^3 + 2x$ is shown on the right. Draw the graphs of $y = f(-x)$ and $y = -f(x)$ on the same axes.



To test if a function is odd, you can test algebraically to see if $f(-x) = -f(x)$.

Example 4

Is the function $f(x) = x^5 + 3x^3 + 5x$ odd?

Note: A polynomial function involving only odd powers of x will be odd.

Supplemental Worksheet #1

1. State whether each of the following is even, odd, or neither.

   a) $f(x) = 3x^2$

   b) $f(x) = -4x^2 + 3x$

   c) $f(x) = |3x|$

   d) $f(x) = 7$

2. How can you test whether or not a function is symmetric with respect to the *y*-axis?

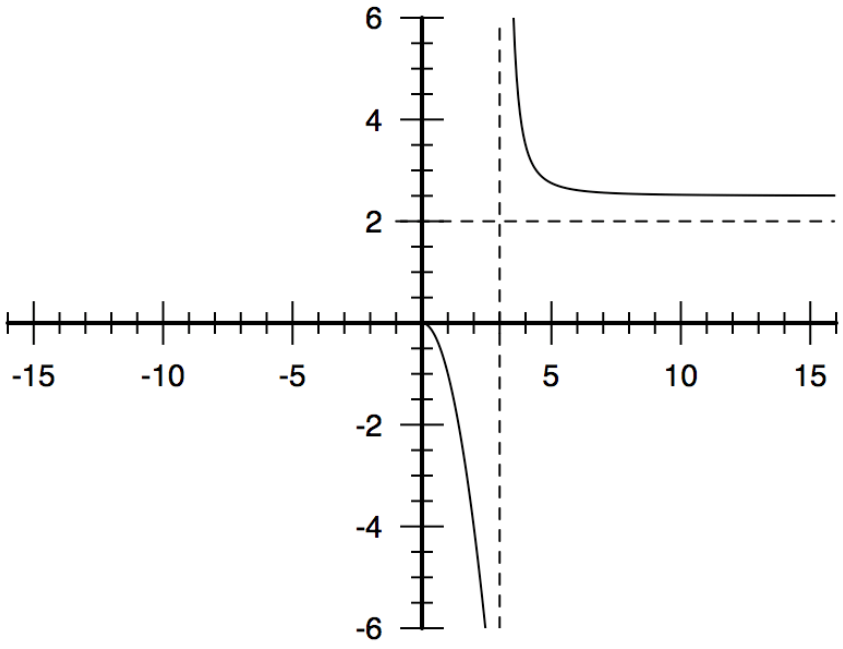
3. For each of the following equations, indicate whether the graph is symmetric with respect to the *y*-axis.

   a) $y = x^2$

   b) $y = \sqrt{x}$

   c) $y = x^4 + x^2$

4. The graph shown below is part of an EVEN function. Complete the graph.



5. The graph shown below is part of an ODD function. Complete the graph.